Are conservation laws metaphysically necessary?

Abstract

Are laws of nature necessary, and if so, are all laws of nature necessary in the same way? This question has played an important role in recent discussion of laws of nature. I argue that not all laws of nature are necessary in the same way: conservation laws are perhaps to be regarded as metaphysically necessary. This sheds light both on the modal character of conservation laws, and on the relationship between different varieties of necessity.

1 Introduction

In his paper *The Varieties of Necessity* (Fine 2002), Kit Fine considers in passing the idea that we might have to distinguish different kinds of necessity among laws of nature. The goal of my paper is to pursue this suggestion further to see whether there are indeed varieties of necessity among laws of nature, and in particular, which laws of nature might be *metaphysically* necessary.

I shall look only at laws of physics, for two reasons. First, it is fairly uncontested that there are laws of physics[1]; second, if we can find a variety of necessities already just among laws of physics, it seems we have already shown that there is more than one kind of necessity to laws of nature, even independent of the question how laws from other sciences might differ from those of physics. Furthermore, there is an independent motivation to look into the possibility of a variety of necessity in the case of laws of physics, since within the philosophy of physics there is a debate about the relationship between conservation laws and symmetry principles on the one hand, and more 'standard' laws of physics on the other, and some philosophers have suggested that conservation laws and other laws might differ in the degree to which they are necessary.[2]

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[1] Of course there are philosophers who object to the idea that there are laws of nature at all, including laws of physics. For the purposes of this paper I will not enter into the debate as to whether there are laws of nature, but simply assume that at least as far as physics is concerned, there are laws, and that the modal status of these laws is a topic of interest to philosophers.

[2] Marc Lange, for example, takes symmetry principles to be meta-laws, and conservation laws to be more necessary than other kinds of laws.(Lange 2009)

2 Necessity - One or Many?

Before we attempt to understand the potentially diverse forms of necessity among laws of physics, it will be helpful to get a better grip on the options for varieties of necessity, that is, on the relationship among logical, metaphysical, and natural necessity.[3]

Pluralism about necessity can be achieved in two different ways, which I will call the 'degree view' and the 'species view' respectively. The difference between a degree view and species view is that on a degree view, what it means to be necessary for a proposition is the same for all propositions: they are all necessary in virtue of possessing a certain feature. But unlike monism about necessity, a degree view of necessity allows that the feature is possessed in different amounts. On a pluralistic view that takes necessity to come in different kinds, by contrast, it is not a single feature that makes propositions necessary, but different ones. Schematically one might put it this way: on a degree view, a proposition is necessary in virtue of possessing feature N, and distinctions are to be made between propositions that are more or less N/possess N to a greater or smaller extent. On a kind view, by contrast, what makes a particular proposition necessary is possession of feature N, and what makes a different proposition necessary is possession of feature N'.

The idea that there is only one form of necessity was especially popular in the

[3] There are other forms of necessities, such as ethical, deontic, and epistemic necessities, which are often thought to be yet more species of necessity. I will set those aside for the purposes of the discussion.

first half of the twentieth century, when logical positivists tried to argue that all necessity was really logical necessity. Logical necessity, at least in the narrow sense envisioned by the logical positivists, is too narrow to comprise all propositions we count as necessary.

One strategy for allowing for propositions other than the narrowly logical propositions to be necessary is to make them necessary *relative* to some set of truths. If certain propositions are 'held fixed' then certain other propositions will be necessary. This relativization strategy has been subject to criticism, and Fine clearly thinks there is no future in it (Fine 2002, 255). Such relative necessity seems too easy to achieve, and hence trivial or insubstantial, since depending on what we are willing to build into the class of propositions 'held fixed', any proposition could potentially turn out to be necessary. If we attempt to define natural necessity in terms of metaphysical necessity, say by suggesting that propositions are naturally necessary if and only if they follow from the laws of nature, we need to show how the resulting 'necessity' is not just a trivial necessity of the form: anything that is logically entailed by some preferred set of propositions is necessary relative to those propositions (Fine 2002).

To develop a degree view of necessity, a more promising strategy is restriction. Instead of starting with the most narrow (or strictest) form of necessity, the second strategy is to start with the broadest notion of necessity and to define the other notions of necessity by restricting them in some way. The typical suggestion for such a broader notion is of course metaphysical necessity, but there are those who suggest that the broadest form of necessity is in fact natural necessity. One such view is Marc Lange's view (Lange 2009).[4] This strategy leads to a degree view of necessity, where different kinds of necessity are related to one another by being restrictions of one another. Logical necessities are a subset of metaphysical necessities, which in turn are a subset of natural necessities.

Kit Fine, by contrast, argues that there are varieties of necessity, and that they are characterized by distinct features, which on my classification scheme makes his view a species view. For Fine, metaphysical necessity is "the sense of necessity that obtains in virtue of the identity of things"(Fine 2002, 254). Fine's suggestion that there might be varieties of necessity among laws is motivated by the idea that not all natural necessities are also metaphysical necessities. This makes Fine's view especially interesting to philosophers of physics, since it suggests not just that some laws of nature might be more general than others, but that in virtue of what laws of nature hold might be different for different laws. Some laws of nature might turn out to hold just in virtue of what happens to be the case in our world, whereas others might turn out to hold in virtue of the identity of things.

To get a better grip on what Fine has in mind, let's look at his way of introducing the idea:

> That electrons have negative charge, for example, strikes one as

[4] Lange's official goal is to argue for a species view of necessity, but he doesn't distinguish between degree views and species views. The details of his arguments suggest that his view is better described as an attempt to take natural necessity as the broadest notion of necessity and then using the strategy of restriction to arrive at the other forms of necessity.

> metaphysically necessary; it is partly definitive of what it is to be an electron that it should have negative charge. But that light has a maximum velocity or that energy is conserved strikes one as being at most naturally necessary. It is hard to see how it could be partly definitive of what it is to be light that it should have a given maximum velocity, or partly definitive of energy that it should be conserved. (Fine 2002, 261)

It is perhaps tempting to understand 'partly definitive' to mean something like: having negative charge is part of the ordinary meaning of 'electron'. To do so would render the proposition 'Electrons have negative charge.' a conceptual or perhaps even linguistic truth. That is not how we should understand Fine here. What he has in mind are 'real definitions' (Fine 1994) – the idea that what really makes a thing *that* thing is not up to our linguistic or conceptual choices. Since having negative charge is partly definitive of electrons, 'that electrons have negative charge' is a better candidate for a metaphysical necessity than that there is a maximum speed, or that this speed should be the speed of propagation of light.

For the remainder of the paper I will accept Fine's notion of 'metaphysical necessity', to see where it leads us. Is Fine right to claim that some laws of nature are metaphysically necessary in this sense, and should we follow his assessment as to which laws those are?

3 Metaphysical Necessity and Laws of Nature - Two Cases

Let us begin with the example Fine offers as a case of metaphysical necessity: 'Electrons have negative charge.'. While it is a bit odd to say that this is a *law* of physics, Fine's intuition that this is partly definitive of electrons seems to be correct. If we found a particle that had the same mass as electrons and behaved like electrons in every other way, except that it was positively charged, we would not call it an electron, we call it a positron. That is good evidence for taking being negatively charged to be partly definitive of what it means to be an electron. Following Fine that means that the necessity involved in this case is metaphysical necessity: it is part of the identity of this kind of thing, electrons, that they are negatively charged. Particles that are not negatively charged, even if everything else about them is the same, are not electrons. So Fine is right, it is metaphysically necessary that electrons have negative charge.

Notice also, though, that the proposition that electrons have negative charge is not exactly a particularly characteristic law of nature. 'Electrons have negative charge' seems a lot more like 'sisters are female' than like '$F = ma$'. So this might in fact not be a case of a law of nature that is metaphysically necessary, but an example of a metaphysically necessary truth that happens to be about certain kinds of particles, but is not thereby any more a law of physics than the proposition that sisters are female is a law of human biology. If so, we have not yet found a law of nature that is metaphysically necessary, we have just found that there may be metaphysically necessary propositions concerned with kinds of entities important for physics. So far at least it is not clear at all what the relationship between such metaphysical necessities and laws of nature might be. Let us look at a different candidate for natural laws, then, conservation laws. Conservation laws are often accorded a special status of some kind by physicists and philosophers of physics. Fine suggests that such laws are at best naturally necessary, but not metaphysically necessary, since it is "hard to see how it could be partly definitive of energy that it should be conserved"(Fine 2002, 261). I take it Fine means for this to hold of other conserved quantities, such as momentum and charge, as well. To deny that conservation laws are metaphysically necessary means to deny that it is part of 'the definition' or a matter of the 'identity' of the various conserved quantities, such as energy, angular momentum, or electric charge, that they are conserved. But even if this might 'intuitively' seem right, it is not clear that it is actually correct. Indeed, we should expect that the assessment of the modal status of such claims depends on the details of the physical theories in which they occur.

In modern physics conservations laws are closely tied to symmetry principles via Noether's theorems[5], and the exact relationship between symmetry principles and conservation laws is a matter of much debate in the philosophy of physics (Brading and Castellani 2003). Some points of that debate bear quite directly on the question of the modal status of conservation laws, so I will focus on those. I will try to keep technicalities to a minimum, many of the details can be found in

[5] Only certain kinds of symmetries (continuous ones), and not all conservation laws (only exact ones).

the literature.[6]

In slogan form, Noether's first theorem is often said to state that for each symmetry of the equations of motion, there is a conserved quantity. This is of course just a slogan, and we need to be more careful in stating just what the theorem says. In general, Noether's theorems apply only to systems for which the equations can be given a Lagrangian formulation. The symmetries are continuous symmetries, taking solutions of the Euler-Lagrange equations into solutions. What Noether's first theorem shows, roughly, is that for a group of continuous global symmetry transformations (given certain conditions) there are conserved (Noether-)currents.[7] Given certain boundary conditions, the existence of conserved currents implies that there are also conserved quantities, commonly called 'Noether-charges'. Noether-charges are conserved as a matter of necessity, or if you like, by definition. Part of what it is to be a Noether-charge is to be conserved.

None of this yet shows that energy, angular momentum, or electrical charge is conserved by (metaphysical) necessity. For in order to show that any of the usual conserved quantities are conserved we have to show that the conditions for the application of Noether's theorem are fulfilled, and (hence) that the quantity of interest turns out to be a Noether-charge. In the classical case this is done by

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[6] See especially (Brading and Brown 2003) and (Brown and Holland 2004).

[7] Noether's first theorem as she puts it reads: "If the integral I is invariant with respect to a G_ρ ,then ρ linearly independent combinations of the Lagrange expressions become divergences – and from this, conversely, invariance of I with respect to a G_ρ will follow. The theorem holds good even in the limiting case of infinitely many parameters." (Noether 1918); transl. (Tavel 1971).

showing that the Euler-Lagrange equations of motion hold. It is because the Euler-Lagrange equations hold that there is a conserved current, and it is only using a further assumption about boundary conditions that we can make the inference from the conserved current to the conserved quantities (Brown and Holland 2004). The conserved quantities of classical mechanics are Noether charges only because the classical equations of motion are what they are.[8] But whether or not the classical equations of motion hold is something that needs to be established, and there are problem cases.

Since we have no reason to treat the equations of motion as metaphysically necessary (they don't hold in virtue of the identity of things), we have no reason to think that classical conservation laws are metaphysically necessary. Given what the equations of motion are, and that they hold where they do, it is indeed necessary that the conservation laws hold, but that's just a conditional necessity. The connection between the symmetries of the equations of motion and conservation laws is shown by Noether's theorem.[9] That these are the correct equations of motion, however, is a completely different matter. As far as deriving conservation laws using Noether's first theorem is concerned, it seems then we should conclude that while it is a metaphysical necessity that Noether-charges are conserved, it is not a metaphysical necessity that energy, linear momentum,

[8] To put this in terms of conserved quantities being Noether-charges may seem like a rather idiosyncratic way of putting this point, but it will be useful as we try to apply Fine's criterion for metaphysical necessity.

[9] Or shown most clearly by Noether's theorem. That there is a connection between symmetries of the equations of motion and conservation laws had been observed before.

or angular momentum is conserved.

Noether's first theorem is applicable to global symmetries, i.e., symmetries depending on constant parameters, in contrast to local symmetries, which depend on *arbitrary functions* of space and time. Interestingly, however, relativistic field theories like quantum electrodynamics are *locally* gauge invariant, but not *globally*. Accordingly, one should expect the relevant Noether theorem not to be her first, but her second theorem, a possibility that has been much discussed in the recent literature in the connection with the role of gauge symmetry (Brading 2002). Noether's second theorem is concerned with local symmetries, not global symmetries.[10]

Despite this, the standard approach to conservation of electric charge in quantum electrodynamics proceeds via Noether's first theorem. Katherine Brading has argue that while this is approach is correct, it is also 'subtly misleading' (Brading 2002). It is misleading because it obscures the fact that the conservation of electric charge here does not depend on the satisfaction of particular equations of motion, but instead follows from the interdependence of matter and gauge fields. This interdependence can seem to look like a the result of a mere mathematical identity, which would suggest that the conservation law holds in virtue of a mathematical truth, not in virtue of the details of the 'real' physics, that is, the

[10] Noether's second theorem, which she proved in the same paper, is stated as follows: "If the integral I is invariant with respect to a $G_{\infty\rho}$ in which the arbitrary functions occur up to the σ-th derivative, then there subsist ρ identity relationships between the Lagrange expressions and their derivatives up to the σ-th order. In this case also, the converse holds."(Noether 1918) Notice that in this theorem, *arbitrary functions* occur in G.

particular equations of motion.[11]

Katherine Brading and Harvey Brown (2003) try to resist the conclusion that conservation of electric charge and energy respectively are 'merely' mathematical, and point out that we do not usually treat the conservation of electric charge as having 'no physical significance'. As a solution to the apparent problem they suggest that what looks like a mathematical identity can be treated as physically significant (as opposed to merely mathematical) as long as we treat the equation as relating two different fields. Then it becomes a constraint on what the field equations can look like, and that is physically significant. So they suggest that we can take conservation laws as physically significant even where it seems possible to derive them as a matter of mathematics.

This somewhat technical detail turns out to be important for the question of the modal status of conservation laws in such theories. While Brading and Brown almost equate 'empirical' and 'having physical significance', which is then contrasted with the merely mathematical identities that can be derived from Noether's second theorem, it seems that this contrast does not quite capture what really seems problematic about deriving conservation laws in this way. For how we come to know that a particular conservation law holds is one thing, but what seems to be the deeper issue is *in virtue of what* the conservation law holds. In the case of theories with global symmetries, it seemed clear that the conservation laws hold in virtue of the particular equations of motion. We should not think

[11] This potential problem is the main concern of (Brading 2002) and (Brading and Brown 2003), but the debate goes back to the early days of the general theory of relativity and Noether's theorems.

that this means we came to know conservation laws in that case empirically, that is, through observation. But since the details of the equations of motion mattered for the derivation, the modal status of the conservation laws depended on the modal status of the equations of motion. In the case of theories with local symmetries, that no longer seems to be the case. For there we do not require the equations of motion in order for the conservation laws to be derived. Now it seems that the conservation laws hold just in virtue of mathematics. That is perhaps the disturbing feature of theories with local symmetries and the use of Noether's second theorem that is made possible through this. It is not a matter of a priori vs. empirical knowledge of the conservation laws, but a question about in virtue of what the conservation laws hold. Brading and Brown may be right that whether we take conservation laws to be physically significant is a matter of further interpretation, but no matter how we interpret the resulting law, it still looks like it holds in virtue of mathematics alone.

If the conservation laws hold as a matter of mathematics, it all of a sudden seems that conservation laws might be metaphysically necessary after all. For whereas in the classical case we needed particular physical laws (the Euler-Langrange equations of motion) to hold in order for the conservation laws to hold, for the case of field theories with local symmetries, this no longer seems needed. What conservation laws depend on seems itself like a good candidate for a metaphysical necessity: the necessary interdependence of two physical fields. It is in virtue of the nature of these fields, that conservation laws hold in theories with local symmetries.

We may add to these somewhat technical consideration a piece of linguistic evidence. In modern field theories, it is quite common to call 'charges' the 'generators of the local symmetry groups' (Martin 2003), it seems as though we should say that electric charge, for example, is conserved in virtue of what charge is, not in virtue of something else, like the equations of motion. Electric charge is conserved because it is a generator of a particular continuous symmetry group, $U(1)$, and the color charge of quarks is conserved because it is the generator of a different symmetry group, $SU(3)$. This shows up in theories theories with local symmetry principles, which are characteristic of modern field theories, and it marks a genuine change from classical mechanics and its conservation laws. What has changed, however, is not their *epistemic* status, but their *modal* status: they now seem to be metaphysically necessary, they hold as partly definitive of the conserved quantities.

4 Conclusion

It turns out, then, that whether conservation laws are metaphysically necessary depends on the theory in which they occur. *Pace* Fine, it is partly definitive of charge to be conserved, at least in quantum electrodynamics. Fine's general point, that there are varieties of laws among laws of nature, however, seems to be justified. Unlike his own example of a metaphysically necessary law of nature –'Electrons are negatively charged.'– conservation laws are clearly a case of laws of nature. So if conservation laws in certain theories turn out to be true in virtue of the identity of some thing, then we should conclude that some laws of nature are metaphysically necessary in Fine's sense. At the same time, if we are inclined to think that classical equations of motion do not hold with metaphysical necessity, we will have to accept that there are varieties of necessity among laws of nature, and that this variety is indeed a variety in kind, not just degree.

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