Fundamental and Derived Quantities

J. E. Wolff – King's College London

Abstract

A widespread view in metaphysics holds that some properties, perfectly natural ones, have an elite status among properties. As part of a naturalistic approach to metaphysics, it is commonly presumed that science, and in particular physics, will reveal which properties in fact play the role of elite properties. Since properties in physics are often quantitative, this raises the question whether science delivers a distinction between base and derivative quantities that matches the distinction in metaphysics between fundamental and non-fundamental properties. In this chapter I investigate whether laws of nature or systems of units can be employed to arrive at such a distinction, and answer in the negative. The requisite definitional dependencies characteristic of fundamental and less-fundamental properties hold among the dimensions of quantities, not the quantities directly. Yet, the decision about which dimension have priority derives from a distinction of quantities into base quantities and derivative quantities that is ultimately conventional.

1. Introduction

It is fairly standard in contemporary metaphysics to distinguish between fundamental and non-fundamental properties. As I will be using these terms here, this distinction is meant to capture David Lewis' idea that some properties are *perfectly natural*, and that these elite properties are what make for objective similarity among objects, as well as doing all kinds of other metaphysical heavy lifting. In order to perform all these metaphysical duties, fundamental properties are usually said to be *intrinsic* and *undefined*; they are the properties in terms of which less natural properties are defined, but not vice versa (Lewis 1983). In keeping with the aim of naturalistic metaphysics, the expectation is that science will reveal to us, which properties are fundamental. Standard candidates for fundamental properties on this naturalistic conception are properties like mass and charge, which feature prominently in physical theories.

In this chapter I would like to address a tension between the features attributed to fundamental properties and the naturalistic desideratum that fundamental properties are discovered by science. Several such tensions have been noted in the literature, most of them having to do with the fact that properties in physics tend to be quantities. Among the known difficulties are the observation that if we use a determinable/determinate model of quantitative properties, it seems that we need to choose whether it is determinables or the determinates that are the fundamental properties, since accepting both as fundamental would seem to be redundant (Hawthorne 2006). Another concern regarding standard candidates for fundamental properties in physics is that it is far from obvious that these properties are genuinely intrinsic. Especially in the context of quantum mechanics, it has been called into question whether the relevant physical properties are indeed intrinsic (Esfeld 2014). Depending on the exact metaphysics of quantities one adopts, one might also worry that quantities themselves are inherently relational, which might further undermine the claim that all fundamental properties are intrinsic (Eddon 2013).

Finally, it has been noted that a standard assumption regarding fundamental properties and their role in laws of nature might be mistaken. It is commonly assumed that only fundamental properties may occur in laws of nature (Armstrong 1983; Lewis 1983). This assumption has recently been challenged by Hicks and Schaffer (Hicks, & Schaffer 2015). They argue that laws, including fundamental laws, contain both fundamental and non-fundamental properties, contra the standard assumption that only fundamental properties qualify to play a role in fundamental laws. Hicks and Schaffer conclude from this not only that the assumption of a close connection between laws and fundamental properties is false, but also that this speaks in favour of a Humean treatment of laws as mere summaries.

For the purposes of this chapter, I'll set aside the first two known issues, and take Hicks' and Schaffer's argument as my starting point. While I agree with their observation that not all quantities in (fundamental) laws qualify as fundamental properties, my diagnosis of the source of the difficulty will be different. As I will show, Hicks' and Schaffer's argument crucially relies on distinguishing between fundamental and non-fundamental quantities, and on aligning this distinction closely with the distinction between fundamental and non-fundamental properties. This alignment requires that fundamental quantities are undefined, while defining less fundamental quantities. I will then argue that there are different ways we might distinguish between more and less fundamental quantities in science, but that none of these distinctions closely matches the distinction between fundamental and non-fundamental properties as required by metaphysics. Especially in the context of systems of units, where the differentiation of quantities into base quantities and derived quantities is particularly explicit, we find that the distinction is in an important sense conventional and non-unique. Since the distinction between fundamental and non-fundamental properties is meant to be metaphysically significant, not merely conventional, the distinction between fundamental and non-fundamental quantities fails to map onto the metaphysical distinction among properties. Furthermore, definitional dependencies hold not between quantities directly, but between the dimensions of quantities; yet the priority of certain dimensions derives from the distinction between base quantities and derivative quantities. In light of this, I conclude that the distinction between fundamental and non-fundamental properties, at least as far as physical quantities are concerned, is less naturalistic than previously thought.

2. Fundamental Properties and Laws

2.1. The broken link between laws and fundamental properties

Since the properties occurring in laws of physics tend to be quantities, a close link between fundamental properties and physical laws suggests that some quantities are (candidates for) fundamental properties in the metaphysician's sense. As we shall see, not all quantities are candidates for fundamental properties: quantities that are defined in terms of other quantities are unlikely to qualify as fundamental quantities, and hence unlikely to qualify as fundamental properties. Since a common condition on fundamental properties is that they are the ones in terms of which other properties are defined, but not vice-versa, being defined at least prima facie counts against a quantity's being fundamental. Metaphysicians have usually turned to laws of nature to find out which quantities are scientifically fundamental. A common guiding thought has been that quantities that occur in fundamental laws of nature are good candidates for fundamental quantities, and indeed, for being fundamental properties. This thought has often been turned into a requirement, namely that "only fundamental properties can be invoked in fundamental laws"[1].

It has recently been argued (Hicks, & Schaffer 2015), that this requirement may be too strong of a connection between laws and fundamental properties, since many laws invoke not only potential candidates for fundamental properties, but also apparently derivative quantities. For example, *F=ma*, a plausible historical candidate for a fundamental law of physics[2], at least prima facie invokes force, mass, and acceleration. Of these three quantities, however, only mass has a claim to being a candidate for a fundamental quantity in the sense intended by metaphysics, argue Hicks and Schaffer.

For the force invoked in the law is the net force acting on a physical body, which is in turn the result of any number of contributing forces. If we are to think of force as a fundamental quantity at all, the contributing forces have a better claim to being fundamental than the net force, even though the latter occurs in a fundamental law. Acceleration, on the other hand is defined as the change in velocity over time, with velocity in turn defined as change of position over time. Since a standard expectation for fundamental properties is that other properties are defined in terms of them, but not vice-versa, it seems that acceleration is not a plausible candidate for a fundamental quantity either (Hicks, & Schaffer 2015). That fundamental properties are undefined, while some quantities in fundamental laws seem to be defined, hence plays a key role Hicks' and Schaffer's criticism of the link between fundamental laws and fundamental properties. To assess this criticism, we need to get a clearer sense of what it means for a quantity to be defined.

Hicks and Schaffer argue that force and acceleration are not fundamental quantities, because they are defined in terms of other quantities. The definitions in question are given by further equations: in the case of acceleration, $a=dv/dt = d^2x/dt^2$. The intended reading is that acceleration is defined as change of velocity over time, not just that the value of acceleration changes as the value of the derivative of velocity over the derivative of time. The relation needed for this reading would be that there is a definitional dependency of acceleration on velocity and perhaps on position. The equation itself does not really yield this definitional dependency, since we can calculate the value of any one quantity given values for the other two (more on this later), so we need to appeal to other considerations. Hicks and Schaffer appeal to the role played by position in Newtonian mechanics (Hicks, & Schaffer 2015, p. 17). Newton's conception of motion is tied to his conception of absolute space and absolute position, yet absolute positions cannot be recovered from accelerations alone. It is on these grounds that Hicks and Schaffer conclude that the definition of acceleration in terms of

[1] This is the formulation offered by Hicks and Schaffer (Hicks, & Schaffer 2015), who then go on to criticise this link between fundamental laws and fundamental properties.

[2] One epistemological concern about the link between fundamental properties and fundamental laws is that it seems to require that we can independently assess the fundamental status of laws and the fundamental status of properties, at least if the link between them is to be a substantive requirement. That is to say, we need to be able to identify candidates for fundamental laws independently of whether they invoke fundamental properties, and we need to be able to say something about whether a property has a claim to being fundamental independently of whether it occurs in a fundamental law.

position and time is irreversible. They similarly argue for the irreversible dependence of resultant forces on component forces by arguing that the latter cannot be recovered from the former alone.

They contrast these cases with cases of reversible definitional dependence, as exhibited by $\rho = m/V$. Density is defined as mass over volume, yet mass can in turn be defined as density times volume. This definition, according to Hicks and Schaffer (Hicks, & Schaffer 2015, pp. 22 note 15) is reversible. While no explicit argument is given, using the recovery criterion employed above, we might argue in the following way: from any two of density, mass, and volume, the remaining third can be calculated without additional information. This seems to contrast with acceleration and net forces, where additional information is needed to recover the absolute position of a body, or the component forces acting on a body. Merely knowing the net force or the acceleration is not enough (indeed, in the case of absolute position, nothing is enough).

Upon inspection, however, the contrast becomes less clear, especially in the case of acceleration. The problem with acceleration arises if we take seriously the idea that Newtonian motion is to be understood as motion with respect to absolute space. Yet this is not the understanding of motion and acceleration in neo-Newtonian theories, which might seem more relevant for evaluation given that Newtonian absolute space is no longer considered a live part of physical theorising.[3] The concern is not that *F=ma* is not a plausible candidate for a fundamental law of nature, which I'm happy to grant. But from any two of *F*,*m*, and *a*, we can calculate the value of the third, just as we can in the case of $\rho = m/V$. The problem for acceleration arises only if we add the Newtonian interpretation of acceleration as a change in motion, and of motion as motion with respect to absolute space. But as the further development of physical theorising shows, these interpretations are optional, and arguably not recommended in light of more recent theories.

Once we take into account further interpretations of quantities, however, volume is not a fundamental quantity either: it is defined in terms of the relevant lengths of the body in question. Since the volume formula depends on the shape of the body, the relevant lengths cannot be recovered from mass and density alone, even though volume can. Using the kind of recovery principle employed by Hicks and Schaffer, then, even $\rho = m/V$ begins to look irreversible. We could either conclude that there are far fewer reversible definitions in physics than we might have thought, or we could reconsider the recovery principle.

Given that $\rho = m/V$ has been used both to define density (as done in the *SI*, for example), and to define mass (as historically done by Newton in the Principia)[4], denying that the definition of density and mass is reversible would seem to go against scientific practice. Of

[3] Hicks and Schaffer acknowledge this point, but set it aside as not central to their argument. I agree that the problem is not whether F=ma is still a fundamental law of physics; but it does seem to matter that the intended priority is not a matter of the equation alone, but requires a further interpretation of the quantities in question. It is possible to reject these further interpretive steps while maintaining that F=ma is law.

[4] "Quantity of matter is a measure of matter that arises from its density and volume jointly" (Newton et al 1999, pp. Definition 1).

course, one might modify the sense in which the definition is reversible: given the volume, either the mass or the density can be calculated, given a value for the other quantity. While this avoids the problem of recovering length information, since the volume is simply assumed, it seems to me that it doesn't quite capture what was meant when we thought about definitional dependencies among quantities. We can reformulate the equation in a variety of different ways to calculate the values of either mass, density, or volume from the other two, yet it would seem strange to suggest that volume is some sense defined as mass over density. Not because volume is a primary or fundamental quantity, but because volume, if defined, is defined in terms of length, not mass or density. As I will argue below, this is largely due to a prior commitment to the dimensions of a quantity, and a commitment to the priority of some dimensions over others.

It seems to me that we should instead reject the recovery principle at work in the arguments by Hicks and Schaffer for the non-fundamental status of acceleration and force. We need to distinguish between our ability to calculate the value of a quantity given the values of other quantities from the question of whether a quantity is defined. Whenever we have an equation relating several quantities, we can calculate the value of one of them from the values of the others. This is unsurprising and it is best understood in terms of a symmetrical supervenience of the values of the quantities, not definitional dependence.

2.2. Supervenience and Equations

A property *A* supervenes on a property *B* if and only if whenever there is a change in *A*, necessarily there is a change in *B*. Supervenience can be symmetrical or non-symmetrical. Let's see whether supervenience might be a good way of describing the relationship between quantities in an equation like $F=ma$ or $T=1/2mv^2$, which prima facie seem to involve defined and undefined quantities. These equations express the idea that the value of force equals the value of mass times the value of acceleration, and that the value of the kinetic energy of a particle equals the value of its mass times the square of the value of its velocity divided by 2, respectively. Since equality of values requires that any changes in the value of force will be matched by changes in the values of mass or acceleration, this strongly suggests that supervenience might be the appropriate relation here. Two questions need to be answered as we look more closely at this relationship. First, is the relation symmetrical or asymmetrical? Second, do the equations have the right modal force?

Since equality is a symmetrical relation, any metaphysical interpretation of an equation will have to respect that the relationship displayed is symmetrical. Supervenience can be symmetrical, so this constraint does not rule out supervenience as a suitable candidate for the relation in question. Any change in mass or acceleration cannot happen without a change in force, and vice versa. So, if force supervenes on mass and acceleration, the same is true conversely. Given that we usually identify force with causal influence, we might be inclined to read changes in the values as changes in force, which then cause changes in acceleration. But this causal interpretation leaves the supervenience relation untouched. The supervenience holds either symmetrically, or not at all.

Supervenience claims can be made with different modal force, depending on how the necessity of the claim is understood. If the necessity in question is natural necessity and the equation expresses a law of nature, then the supervenience claims holds. It is not possible for the value of force to change without a change in the value of mass or acceleration without violating a law of nature. The same is not true if the necessity is meant to be metaphysical necessity, if metaphysical necessity is distinct from natural necessity. If laws of nature can vary across different metaphysically possible worlds, then there could be worlds in which the value of force can change even without a change in the value of mass or acceleration. These would be worlds in which $F=ma$ is not a law of nature. If we want to interpret the relationship between values of quantities in equations relating them as a form of supervenience, the modal force of this supervenience claim must be understood as natural necessity, not metaphysical necessity.

Suppose for the moment that this is how we should metaphysically interpret the equations in question. Does interpreting the relationship in these equations as a form of supervenience help with establishing which quantities are fundamental? It seems not. For as we've seen, as far as the equations themselves are concerned, the relationship is symmetrical. What we had been looking for, though, was a relationship that would clearly indicate one quantity as metaphysically prior to the other quantities in these equations. Instead we find that while supervenience fits as a characterisation of the relationship in question, it does little to establish the desired priority. This might suggest that supervenience in general is not quite the right relation we should be looking for, but it even more clearly suggests that the equations relating quantities are not by themselves sufficient to establish the priority relation in question.

The supervenience relations in these equations do not guarantee that we can recover the values of further defining quantities from these equations, as the cases of acceleration and kinetic energy show. This lack of recovery is a feature, not a bug, as is especially apparent in the case of kinetic energy. Kinetic energy is a useful quantity, precisely because we do not need to know (and, in many cases, don't know) the exact values for the masses or velocities of various particles. But if we know their kinetic energy, we can nonetheless make further calculations about their behaviour. Similarly, we might be able to determine the acceleration of a body even though we are not able find out its absolute position. In this case, the ultimate conclusion we've drawn is that there are no absolute positions. An even more striking example is electric field strength and electric potential. The strength of an electric field at a point, mathematically, is the negative gradient of the potential, and the exact value(s) of the potential cannot be recovered from the field strength alone. Yet the electric field is typically regarded as a bona fide physical quantity, whereas the potential is regarded as a mere mathematical quantity. The reason is that absolute values for potentials make no observational difference; only differences between potentials make an empirical difference.[5] The case of acceleration compared to absolute position, and to some extent even compared to velocity, is quite similar. While we can of course locate bodies at relative positions and describe their relative motions, any absolute position, and indeed, following relativity theory, any absolute motion, remain elusive.

Our inability to recover particular values for quantities like absolute position or absolute potentials from the relevant laws does not indicate that these quantities are more

[5] As was vividly explained by Maxwell in his elementary treatise on electricity (Maxwell 1881, pp. 53-54).

fundamental; on the contrary, it might indicate that the quantities are not physically significant, because they fail to make an empirical difference.

2.3. Intermediate Conclusion

Hicks' and Schaffer's observation that defined or derivative quantities occur in laws of nature is correct, but the further conclusions regarding fundamental properties in laws seem problematic. This argument relied on importing the idea that the less fundamental is defined in terms of the more fundamental from the metaphysics discussion to the situation in physics. As we've just seen, however, it is quite difficult to say, exactly when physical quantities are defined in terms of one another, and even where such definitions are offered, it is not clear that the ostensibly defining quantities qualify as more fundamental, all things considered. Indeed, as the examples of acceleration and electric field strength suggest, quantities that make an empirical difference may count as more fundamental, even when they were originally defined in terms of quantities that do not. This suggests that definitional relationships between quantities are difficult to establish from laws alone, and that considerations from physics might speak against taking apparently defined quantities to be less fundamental than their definiens.

I will have more to say about definitions of quantities in what follows, but for now I note that laws are not as straightforward a source of fundamental properties as one might have hoped. An alternative strategy is to look at metrological distinctions among quantities for help. Bradford Skow (Skow 2017) has recently suggested that there might be a connection between the differentiation into base quantities and derivative quantities (or primary and secondary quantities) commonly used in metrology, and fundamental properties in the metaphysician's sense. If primary or base quantities in the metrological sense are fundamental in the metaphysician's sense, this would offer another way of reading off fundamental properties from scientific theories. Indeed, prima facie it would also offer a way of finding fundamental properties without having to detour through laws at all, thereby circumventing the problem of having to determine which laws ought to count as fundamental. To assess the viability of this strategy, we need to take a closer look at the way in which metrologists draw the distinction between base quantities and derivative quantities.

3. Systems of units and the distinction of quantities into base and derived quantities

3.1. Are the base quantities of the SI fundamental?

Metrologists introduce a distinction between base and derivative quantities in the context of setting up a system of units of measurements. The most widely used system of this kind is the *International System of Units (SI)*, with the base quantities length, time, mass, thermodynamic temperature, electric current, amount of substance, and luminous intensity (SI, 2006). In the *SI*, these base quantities are given with the following base units: the metre, the second, the kilogram, the Kelvin, the Ampere, the mole, and the candela. From these, all other quantities and their units are defined. Several points are worth noting about this list of base quantities.

The first observation is that the *SI* does not strive for minimality of base quantities. As the name suggests, the *SI* is an international collaborative effort, and it serves the needs of engineers and scientists, as well as commercial and industrial users. A metaphysician might suspect that this wide audience makes the whole system too practice oriented, and might feel inclined to reject some of the base quantities as not being genuinely fundamental. Amount of substance, for instance, which was only included in the *SI* in 1971, might well be rejected as a fundamental quantity by metaphysicians, given that it is treated with suspicion even by many physicists (it is important in chemistry, though). How wide of a base of quantities one is willing to tolerate might depend on one's views about reductionism and the role of physics vis-à-vis the "special" sciences. Should a system of units only include base quantities from physics? And if so, are we to include only "fundamental" physics, like mechanics, or also astronomy?

A second point to note is that while some quantities listed as base quantities in the *SI*, like length, time, and mass, correspond to intuitively fundamental quantities,[6] in the case of others, like electric current, there seems to be a more plausible alternative candidate for fundamental quantity, namely electric charge. After all, if current is just charge per unit of time, why should current be more fundamental than charge? Historically the reason to include current rather than charge seems to have been that it is easier to realise, with arbitrary precision, units for current than units for charge. This would seem to be exactly the kind of practical consideration metaphysicians might regard as misleading when it comes to determining which quantities are fundamental. As we shall see below, in the context of the 2018 reform of the *SI*, this matter becomes even more difficult to decide, since the proposed new definition of the Ampere will now make reference to the fixed numerical value of elementary charge. While current remains a base quantity in the *SI*, charge is thereby brought in indirectly in the definition of the unit. We will need to look more carefully at the role of the different quantities below.

A third concern is that while the *SI* "conventionally" stipulates the base quantities to be mutually independent, there are interdependencies in the definitions of their units. For example, the definition of the mole—the unit of amount of substance—uses the kilogram, and the definition of the metre involves the speed of light and thereby the second. The exact interdependencies will to some extent be reshuffled in the proposed reform of the *SI*, but there remain interdependencies even so.[7] Indeed, the new definitions of the units will involve constants, some of which bring in quantities that are not themselves among the base quantities of the *SI* (as mentioned above, the new definition of the Ampere will involve electric charge). This suggests on the one hand, that we should not take the independence of the base quantities from one another in the *SI* to be indicative of their status in physical theorising—physics may very well regard some of them as heavily dependent on others. On the other hand, it suggests that in assessing the definitional independence of quantities, we need to distinguish between a quantity itself being (definitionally) independent of other quantities, and its unit being definitionally independent. The former might obtain even if the latter does not.

---

[6] Whether all of length, time, and mass genuinely qualify as fundamental in light of current physics is a further question.

[7] According to the new definitions, the mole will no longer invoke the kilogram, but the kilogram in turn will involve the second, for example.

All of this should give us pause when pursuing the strategy of determining which quantities are fundamental by starting from the distinction between base quantities and derived quantities. We need to understand better what goes into setting up a system of units to understand when we might be in a position to take its base quantities to be fundamental in the relevant metaphysical sense.

3.2. What does it take to set up a system of units?

To set up a system of units, "it is necessary first to establish a system of quantities, including a set of equations defining the relations between those quantities" (SI, 2006, p. 103). The relevant equations are drawn from standard physics and engineering textbooks. As examples the *SI* offers $F=ma$, relating force, mass, and acceleration, or $T=1/2\ mv^2$, which relates the kinetic energy of a particle to its mass and velocity. There are of course many more such equations, which are summarised in a separate publication, the international standard *ISO/IEC 80000: Quantities and Units*. Within this system of quantities, a distinction is then drawn between base quantities and derived quantities, and it's worth noting that metrologists tend to emphasise that this distinction is conventional.[8] The small set of base quantities is then associated with corresponding base units; while the base units are defined in the *SI*, the base quantities are typically left undefined (amount of substance is somewhat of an exception, since the *SI* provides a definitional gloss for it, unlike for any of the other quantities). The equations relating the different quantities are needed to guide the introduction of derived units for the non-base quantities in the system of units. The units for derived quantities are typically arrived at by replacing the quantity equations with "unit-equations", in which the familiar quantity symbols are replaced by the standard units of the base quantities. For example, $T=1/2\ mv^2$ becomes $Joule = kg\ m^2/s^2$.[9]

The requirements for setting up a system of units make it clear very quickly that looking to metrology does not avoid reference to laws altogether. After all, the equations relating quantities will typically be laws. What is interesting, however, is that the laws are not there to identify the base quantities, since those are determined by convention, but rather to relate the derivate units to the base units in a coherent fashion. This makes it clear that if metaphysicians wish to draw on the base quantities of systems of units as candidates for fundamental quantities, they need to accept that laws of nature relate fundamental and non-fundamental quantities, instead of involving only fundamental quantities. Hicks' and Schaffer's observation that fundamental laws sometimes contain derivative quantities begins to look like a standard feature of laws, if we assume that the relevant sense of derivative quantity is given by metrology.

The emphasis metrologists place on the conventionality of the choice of base quantities is due to the fact that different classes of systems of units employ different quantities as base quantities. For example, while the *SI* recognises seven base quantities, the Gaussian system

---

[8] "From a scientific point of view, the division of quantities into base quantities and derived quantities is a matter of convention, and is not essential to the physics of the subject" (SI, 2006, p. 103).

[9] More precisely, each base quantity has an associated base dimension and an associated base unit. The relations between base units and derived units are strictly speaking relations between dimensions, not units. More on this in section 4 below.

of units only recognises three base quantities—length, time, and mass—and standardly uses centimetre, second, and gram as their base units. Units for other quantities, including electromagnetic quantities, are then defined in terms of these three base units. While conversion from Gaussian to *SI* units is straightforward for mass, length, and time, which occur as base quantities in both systems, the conversion of units treated as derived in the Gaussian system to units treated as base units in the *SI* is more complicated. The Gaussian system and the *SI* belong to two different classes of systems of units, because they differ in their base quantities. By contrast, a predecessor of the *SI*, the metre-kilogram-second (MKS) system, and the centimetre-gram-second system belong to the same class, since both recognise the same three base quantities: length, mass, and time. While the *SI* is the most widely used system of units, there is no reason based on physical theorising alone for preferring the *SI* over the more minimalistic Gaussian system or yet other systems of units based on yet different base quantities. Favouring the *SI* is broadly speaking a practical matter: it's widely used, convenient, and satisfies a wide range of users beyond physics.

3.3. Is there a preferred system of units?

Given that the distinction into base quantities and derived quantities makes sense only relative to a particular system of units, it seems that the question of whether or not base quantities can be identified with fundamental quantities comes down to the question of whether there is a preferred system of units. If there were, then it would be natural to identify the fundamental quantities with the base quantities of the preferred system, and to set aside the base quantities of alternative, non-preferred systems.

Skow suggests that to find the relevant base quantities, we should look to systems of units used by scientists when solely concerned with the formulations of laws (Skow 2017, p. 181). The contrast is meant to be with situations in which computational ease determines the choice of units, and perhaps also with cases where quantities are chosen as base quantities because their units are easier to realise as measurement standards (as in the case of electric current). It's not clear, though, that the purpose of being concerned with the formulation of laws is specific enough to single out a unique preferred system of units. As we've seen, some laws are presupposed when setting up the system of units, to relate quantities to one another. This is vital for constructing derived units from base units in a systematic fashion. Choosing a system of units does not proceed independently of what the intended system of laws is supposed to be. More importantly, since the laws relate base quantities and derivative quantities, even if scientists are only concerned with the formulation of the laws, the distinction into base quantities and derivative quantities can be made even after the laws have been agreed upon. The same set of laws can give rise to different systems of units using different quantities as base quantities.

Perhaps what Skow is getting at is that we should take seriously base quantities in systems of units that somehow reflect their status in physical theory, not their computational or operational convenience. Doing so might explain why many physicists (and philosophers) would prefer to use eletric charge as a fundamental quantity, not electric current, even though the latter, but not the former is a base quantity in the *SI*. But this means using yet another way of determining, which quantities qualify as fundamental, different from either the laws or the system of units in question. Rather than reading off the fundamental quantities from either a system of laws or a system of units, we need an independent grip on what makes a quantity fundamental in physics, so that we can then judge which quantities in the laws are to count as base quantities in the preferred system of units. Before asking what further criteria we might bring to bear, I will discuss two alternative strategies for using systems of units.

Instead of choosing a single preferred system of units, one might instead focus on quantities that are regarded as base quantities in most systems of units: mass, length, and time. As we saw above, not only the *SI*, but also plausible, currently used alternatives like the Gaussian system of units employ these three quantities as base quantities. So perhaps instead of looking for a single system of units that will be uniquely preferred, we should instead look for quantities that count as base quantities in all, or at least most, systems of units, and take that to be evidence for their fundamentality. An advantage of doing so would be that it avoids inclusion of some of the more doubtful candidates for fundamental properties, like amount of substance. It would also permit us to acknowledge that there is no unique best system of units, or even a class of best systems of units, where the latter differ only in the way the units are set up, but not in the quantities chosen as base quantities. Since we are interested in quantities, rather than units, in searching for a preferred system of units we really only need to find a preferred class of systems of units to decide, which quantities are to count as fundamental.

A major disadvantage of the proposed strategy is that it would increase the likelihood of 'undercounting' fundamental quantities. If we are only willing to accept quantities as fundamental that are counted as base quantities in most systems of units, then this set will inevitably be smaller than the set of possible candidates for base quantities. The comparison between the Gaussian system and the *SI* is a case in point. The *SI* recognises many more quantities as base quantities, yet if we take the Gaussian system to be a serious competitor, we have to restrict ourselves to those quantities common between the two, which in this case means the Gaussian systems 'wins out', since its base quantities are a subset of the base quantities of the *SI*. Any electromagnetic quantities, including charge, would not count as fundamental in this case, which might seem counterintuitive both from the standpoint of physics, and the standpoint of metaphysics, which tends to count charge as a plausible candidate for a fundamental property. However, if we hang on to the idea that fundamental quantities should not be definable in terms of other quantities, then we should strive for minimality. If the Gaussian system can get by with fewer base quantities, this gives us reason to prefer its base quantities to those of the *SI*. If we wanted to include a base quantity for electromagnetic phenomena, we have a choice between charge and current. If our goal is to accept only base quantities common to several systems of units as fundamental, however, we would not be able to choose between these options. Instead we might be facing pressure to accept neither, leading us straight back to the Gaussian system.

Yet another strategy, pulling in a slightly different direction, would be to pick the system of units based on the overall balance of strength and simplicity analogous to the familiar Humean move of taking the axioms of the best axiomatic system to be the laws. This might speak in favour of using the *SI*, rather than the very austere Gaussian system, or it might speak in favour of a mixture of the two, perhaps using charge as a fundamental quantity in addition to length, mass, and time. The problem with this strategy is not only the standard worry that there might not be a unique best system in terms of strength and simplicity. More importantly, it would seem to yield fundamental properties that are themselves no more robust than laws on the Humean picture. While the Humean picture recognises laws of nature and identifies them as the axioms of the best system, the laws are in a sense metaphysically secondary: they supervene on the non-nomic facts. However, if we identify (a subset of) the fundamental properties by deciding on the fundamental quantities on the basis of a best systems style analysis, then it seems the status of (at least those) fundamental properties will be akin to that of Humean laws. At the very least that seems to go against the reasons for introducing fundamental properties in the first place.

It seems therefore, that while there are a variety of pragmatic considerations, including simplicity and strength, that might speak in favour of one system of units or another, there isn't a clear winner, or a clear preferred system of units. As a result, we should be hesitant to identify the base quantities of any given system of units with fundamental properties in the Lewisian sense.

4. Fundamental Quantities and Fundamental Physics

4.1. Conserved quantities as fundamental quantities

From within physics, a different criterion might be offered. Instead of looking for intrinsic properties, physicists might be more interested in conserved quantities. This might explain what is special about charge, in contrast to current. Charge is special not because it is intrinsic, but because charge is a conserved quantity. While there is a bewildering variety of quantities, only relatively few quantities are conserved, which might appeal to those aiming for a minimalist selection of fundamental properties. Yet this proposal, too, has several drawbacks. Whether a quantity is conserved or not is not always easily determined, and may seem to depend on the theory (and its laws) under consideration. In Newtonian physics, mass is conserved, in relativistic physics, only mass-energy is conserved.

A defender of fundamental quantities might respond that we are of course interested only in quantities that are genuinely conserved. We are interested in quantities that are conserved according to our best theory, not in quantities that we used to think of as conserved quantities in the context of a now obsolete theory. Even so, accepting conserved quantities as the fundamental quantities means rejecting some of the standard metaphysical candidates for fundamental properties, like mass. Nor do any of the base quantities of the *SI*, or for that matter the Gaussian system, qualify as fundamental. Finally, it is not clear we can in fact derive all other quantities from conserved quantities only. The laws connecting would-be base and derivative quantities are typically not conservation laws, and whether an equation like *F=ma*, which can be used to set up a system of units, should count as involving conserved quantities depends on the theory in which it occurs. Conserved quantities matter in physics, but they are not the definitional basis from which other quantities are derived. In this way conserved quantities are not too different from quantities that are important because they make an empirical difference, like acceleration or electric field strength. In both cases quantities are important in physics and play an important role in laws, yet they are not the base quantities from which the definitions of other quantities are derived.

4.2. Dimensions and the definitional status of quantities

Mass is not a conserved quantity in relativity theory, and length is not an invariant quantity in contemporary physics, but relative to the state of motion of the observer. Yet both are base quantities in the *SI.* Why is that? Aside from considerations of tradition and practical application, a key reason for according length this special status is that both L and M, the dimensions of length and mass respectively, are very attractive as base dimensions. As I mentioned in an earlier footnote, systems of units associate each base quantity with a base dimension as well as a base unit.[10] These base dimensions can then be used to define the dimensions of derived quantities in terms of the base dimensions. "In general the dimension of any quantity *Q* is written in the form of a dimensional product, dim $Q = L^{\alpha} M^{\beta} T^{\gamma} I^{\delta} \Theta^{\varepsilon} N^{\zeta} J^{\eta}$ where the exponents α, β, γ, δ, ε, ζ, and η, which are generally small integers which can be positive, negative or zero, are called the dimensional exponents" (SI, 2006, p. 105) . This general fact makes it possible to keep the set of dimensions minimal. The units of derived quantities are found by looking at their dimensions, and their dimensions in turn are found by looking at the relations in which they stand to other quantities.[11] In general, a key aspect of setting up a system of quantities and choosing base quantities is to find base dimensions, which are then used to define derived dimensions. Quantities and units proliferate, while the set of dimensions remains minimal.

By expressing a derived quantity's dimension in terms of the dimensional product, a definitional relationship of the quantity whose dimension is being defined on the base quantities is articulated. A quantity definitionally depends on another quantity (or quantities), if its dimensions definitionally depend on the dimensions of these quantities. This helps to shed light on some of the defined quantities discussed in earlier sections of the chapter. The dimension of volume is L^3, that of velocity is LT^{-1}, that of acceleration is LT^{-2}. Unlike the equations among quantities, which as we earlier saw are best understood as supervenience relations, the relations among their dimensions can be understood as relations of definitional dependence. For whereas we can easily reverse equations to calculate the value of whichever quantity we are most interested in from given values for the other quantities, the situation is quite different for dimensional equations. These equations do not relate changeable values, which suggested a symmetric supervenience relationship for equations among quantities. Instead they seem to make explicit the dependence of the dimensions of derived quantities on the base quantities, by showing how the dimensions of derived quantities can be defined in terms of the dimensions of base quantities. Because we have chosen which dimensions will count as the base dim

Now, dimensions look promising as the sorts of entities that stand in definitional dependence relations, which was something quantities by themselves didn't seem to deliver. But it should be noted that at least in standard ways of setting up systems of units, it is the quantities and their systems of equations that provide the starting point, whereas the dimensions and their relationships are secondary. So, it doesn't seem as though we have a way of determining,

---

[10] The base dimensions of the *SI* are as follows: length L, mass M, time or duration T, electric current I, thermodynamic temperature Θ, amount of substance N, and luminous intensity, J.

[11] Interestingly, this epistemic direction can be turned around. Sometimes we might not know the exact form of the equation relating several quantities, but we do know what the units should be. Dimensional analysis can then help determine what the equation must look like.

which dimensions are to count as the fundamental ones, independent of a suitable system of units. Which of course brings us right back to the difficulties considered in the previous section. But at least we've located definitional dependence relations. They don't hold directly among quantities, but instead among their dimensions. We are tempted to say that a quantity is defined in terms of another quantity (or several other quantities), when its dimension is defined in terms of the dimensions of these other quantities. We should resist this temptation, however. One important reason for this is that different quantities can end up having the same dimensions. So instead of saying either that quantities are defined in terms of other quantities, or saying that quantities are defined in terms of their dimensions, we should say that a quantity's dimensions are defined in terms of the dimensions of the base quantity, unless that quantity happens to be a base quantity. Quantities do not stand in definitional relationships, although they do of course stand in a range of other relationships, including supervenience relations.

4.3. Fundamental constants and definitions of units

Base quantities are typically undefined, and even in the case of derived quantities, it is not clear in which sense they are explicitly defined in terms of other quantities. What we have noted instead is that their dimensions are defined as powers of base dimensions. Units, on the other hand, are explicitly defined. More importantly, base units as well as derived units are defined. Traditionally the base units were defined in terms of either prototypes or physical processes. With the 2018 reform of the *SI*, all base quantities will be defined in terms of fundamental constants. How does this affect our considerations regarding fundamental quantities?

We can distinguish between dimensional and dimensionless constants. The constants used to define units in the *SI* are dimensional, which is why they can be used to define units in the first place. Even prior to the current reform, the metre was defined in terms of the speed of light. This was done by fixing the value of the speed of light in vacuum at 299,792,458 metre/second. By fixing the exact value of c in certain units, namely metre and second, the metre can then be defined as the distance light travels in 1/299,792,458 seconds. By fixing the value of a natural constant (in certain units), this approach to defining units avoids reliance on artefacts or prototypes. In the new *SI*, all units will defined in this way, using the following constants: hyperfine splitting of Caesium Δv_{Cs} to define the second, the speed of light in vacuum *c* to define the metre, Planck's constant *h* to define the kilogram, elementary charge *e* to define the Ampere, the Boltzmann constant k_B to define the Kelvin, the Avogadro constant, N_A to define the mole, and luminous efficacy, K_{cd} to define the candela (Ninth Draft Brochure 2015).

It is interesting to note that while in the case of the metre, using a constant to define the unit does not bring in any quantities not already in the *SI*, other constants, like elementary charge, Planck's constant, or the Boltzmann constant seem to do just that. In the case of the latter two, the quantity brought into the definition is energy, with unit Joule. Their units are given in Joule times second and Joule/Kelvin respectively. However, since the dimensions for energy and charge can be expressed in base dimensions of the *SI*, no new dimensions are introduced, even though indirectly additional quantities are involved in the definition of the units.

The considerations in 4.2 and 4.3. suggest that at least in the proposed new version of the *SI*, we have a 'competition' of sorts among quantities. On the one hand, there are the official base quantities, whose units and relations determine the definition of all other units. On the other hand, there are quantities that are brought in through the definition of the base units, once we define the latter by means of fundamental constants rather than artefacts. This tension is resolved if we acknowledge that the definitional dependencies among quantities play out in the relationships among their *dimensions*, not in the relationship between the quantities directly. It is the dimensions that stand in definitional dependence relations, although the direction of these dependence relations is determined by a choice of system of units, not independently.

5. Conclusion

We must distinguish between the role a quantity has in contemporary theory, and its status as a base quantity. The former depends on a variety of factors, including whether it is conserved or whether its values make an important empirical difference. The latter is a status a quantity has within a given system of units. If fundamental properties are supposed to define non-fundamental properties, then for the case of quantities this has to proceed through the detour of dimensions, since only dimensions stand in definitional dependence relations to each other. Which dimensions are fundamental, on the other hand, is determined by a choice of system of units, which I have argued is ultimately conventional, even though there are of course pragmatic virtues favouring some systems over others. As a result, I think the distinction between fundamental and non-fundamental properties, when applied to quantities, is not as naturalistic as usually assumed. Neither physical theory nor metrology provide a non-conventional and uncontested criterion for which quantities are to count as fundamental and which as derived. We could of course decide to give up on definitional dependence for fundamental properties. That might allow us to say that certain quantities important in physical theorising are indeed fundamental, even though they are defined in the sense that their dimensions are not among the base dimensions. That might be an attractive solution, but it requires a revision in our understanding of fundamental properties.

References

Armstrong, D.M., 1983, *What is a law of nature?* Cambridge University Press, Cambridge.

BIMP dated 11 Dec 2015, *Draft of the ninth SI Brochure.* Bureau International des Poids et Mesures.

Eddon, M., 2013, Fundamental properties of fundamental properties, *Oxford Studies in Metaphysics*, 8, pp. 78-104.

Esfeld, M. 2014, Physics and Intrinsic Properties, in R Francescotti (ed), *Companion to Intrinsic Properties,* de Gruyter, Berlin, pp. 253-70.

BIMP 2006, *The International System of Units (SI),* Goebel, E., Mills, I., & Wallard, A. eds. Bureau International des Poids et Mesures.

Hawthorne, J., 2006, *Metaphysical essays,* Clarendon, Oxford; New York.

Hicks, M.T. & Schaffer, J., 2015, Derivative Properties in Fundamental Laws, *The British Journal for the Philosophy of Science*, p. axv039.

Lewis, D., 1983, New work for a theory of universals, *Australasian Journal of Philosophy*, 61(4), pp. 343-77.

Maxwell, J.C., 1881, *An Elementary Treatise on Electricity,* Clarendon Press, Oxford.

Newton, I., Cohen, I.B. & Whitman, A., 1999, *The Principia - Mathematical Principles of Natural Philosophy,* University of California Press.

Skow, B., 2017, The Metaphysics of Quantities and Their Dimensions, *Oxford Studies in Metaphysics*, 10.